Name: Key Date:

Math 9 Enriched Section 2.6 Rationalizing Radicals

1. Multiply each of the following radicals with its conjugate: Note: $(a+b)(a-b)=a^2-b^2$

a) $(\sqrt{3}+2)(\sqrt{3}-2)$
$3-4$
$=-1$ //

b. $(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})$
$5-3$
$=2$ //

c. $(2\sqrt{3}-1)(2\sqrt{3}+1)$
$=12-1$
$=11$ //

d. $(3-\sqrt{3})(3+\sqrt{3})$
$9-3$
$=6$ //

e. $(2\sqrt{3}-2\sqrt{2})(2\sqrt{3}+2\sqrt{2})$
$12-8$
$=4$ //

f. $(2\sqrt{5}-6)(2\sqrt{5}+6)$
$20-36$
$=-16$ //

2. Rationalize each of the following expressions:

a) $\frac{1}{\sqrt{2}-\sqrt{3}}$

b. $\frac{2}{2\sqrt{3}+5}$

c. $\frac{\sqrt{2}}{2\sqrt{3}+\sqrt{5}}$

d. $\frac{\sqrt{2}+\sqrt{3}}{\sqrt{3}-\sqrt{2}}$

e. $\frac{\sqrt{10}}{\sqrt{8}-\sqrt{6}}$

f. $\frac{5\sqrt{3}}{2\sqrt{2}-3\sqrt{3}}$

3. Simplify:

a) $\frac{7}{\sqrt{20}}-\frac{4}{\sqrt{12}}$
$\frac{7\sqrt{5}}{2\sqrt{5}(\sqrt{5})}-\frac{4\sqrt{3}}{2\sqrt{3}\sqrt{3}}$
$\frac{7\sqrt{5}}{10}-\frac{4\sqrt{3}}{6}$
$\frac{21\sqrt{5}-20\sqrt{3}}{30}$ //

b. $\frac{3\sqrt{48}}{2\sqrt{75}}-\frac{2\sqrt{24}}{\sqrt{96}}$
$\frac{3(\cancel{4}^{2})\cancel{\sqrt{3}}}{\cancel{2}(5)\cancel{\sqrt{3}}}-\frac{\cancel{2}\cancel{\sqrt{24}}}{\cancel{2}\cancel{\sqrt{24}}}$
$=\frac{6}{5}-1=\frac{1}{5}$ //

c. $\frac{3}{\sqrt[3]{12}}=\frac{3}{\sqrt[3]{3\times 4}}\times\frac{\sqrt[3]{3^2\times 2}}{\sqrt[3]{3^2\times 2}}$
$=\frac{3(\sqrt[3]{18})}{6}$
$=\frac{\sqrt[3]{18}}{2}$

d. $\frac{\sqrt{3}}{\sqrt[3]{36}}=\frac{\sqrt{3}}{\sqrt[3]{6^2}}\times\frac{\sqrt[3]{6}}{\sqrt[3]{6}}$
$=\frac{(3^{\frac{1}{2}})(6^{\frac{1}{3}})}{6}$
$=\frac{(3^{\frac{3}{6}})(6^{\frac{2}{6}})}{6}$
$=\frac{\sqrt[6]{27\times 36}}{6}$
$=\frac{\sqrt[6]{972}}{6}$ //

e. $\frac{x^4+x^2}{\sqrt{x^3}}=\frac{x^4+x^2}{x\sqrt{x}}$
$=\frac{x^3+x}{\sqrt{x}}\frac{(\sqrt{x})}{(\sqrt{x})}$
$=\frac{\cancel{x}(x^2+1)\sqrt{x}}{\cancel{x}}$
$=x^2\sqrt{x}+\sqrt{x}$

f. $\frac{5\sqrt{2}}{6-3\sqrt{3}}\frac{(6+3\sqrt{3})}{(6+3\sqrt{3})}$
$=\frac{30\sqrt{2}+15\sqrt{6}}{36-27}$
$=\frac{30\sqrt{2}+15\sqrt{6}}{9}$
$=\frac{10\sqrt{2}+5\sqrt{6}}{3}$ //

g. $\frac{2}{\sqrt{2}+\sqrt{3}+\sqrt{5}}$ $\frac{(\sqrt{2}+\sqrt{3}-\sqrt{5})}{(\sqrt{2}+\sqrt{3}-\sqrt{5})}$

$= \frac{2(\sqrt{2}+\sqrt{3}-\sqrt{5})}{4+\sqrt{6}-\cancel{\sqrt{10}}+\sqrt{6}+3-\cancel{\sqrt{15}}+\cancel{\sqrt{10}}+\cancel{\sqrt{15}}-5}$

$= \frac{2(\sqrt{2}+\sqrt{3}-\sqrt{5})}{2+2\sqrt{6}} \frac{(1-\sqrt{6})}{(1-\sqrt{6})}$

$= \frac{(\sqrt{2}+\sqrt{3}-\sqrt{5})(1-\sqrt{6})}{1-6} = \frac{(\sqrt{2}+\sqrt{3}-\sqrt{5})(1-\sqrt{6})}{-5}$ //

h) $\frac{5}{\sqrt{2}+\sqrt{5}-\sqrt{7}}$

i) $\frac{\sqrt{10}}{\sqrt{2}+\sqrt{6}-\sqrt{8}}$ $\frac{(\sqrt{2}-\sqrt{6}+\sqrt{8})}{(\sqrt{2}-\sqrt{6}+\sqrt{8})}$

$= \frac{\sqrt{20}-\sqrt{60}-\sqrt{80}}{2-\cancel{\sqrt{12}}+\cancel{\sqrt{16}}+\cancel{\sqrt{12}}-6+\sqrt{48}-\cancel{\sqrt{16}}+\sqrt{48}-8}$

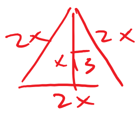
$= \frac{2\sqrt{5}-2\sqrt{15}-4\sqrt{5}}{2\sqrt{48}-12}$

$= \frac{(\sqrt{5}-\sqrt{15}-2\sqrt{5})(2\sqrt{3}+6)}{(2\sqrt{3}-6)(2\sqrt{3}+6)}$

$= \frac{(-\sqrt{15}-\sqrt{5})(2\sqrt{3}+6)}{12-36}$

$= \frac{(\sqrt{15}+\sqrt{5})(2\sqrt{3}+6)}{24}$ $\frac{(\sqrt{15}+\sqrt{5})(\sqrt{3}+3)}{12}$ //

4. The area of an equilateral triangle is $9\sqrt{3}\ cm^2$. What is the number of cm in a side of the triangle?



$\frac{(\cancel{2}x)(x\sqrt{3})}{\cancel{2}} = 9\sqrt{3}$.

$x^2\sqrt{3} = 9\sqrt{3}$

$x = 3$

$\boxed{\text{SIDE LENGTH} = 2x = 6}$ //

5. Evaluate: $x=\sqrt{20+\sqrt{20+\sqrt{20+.......}}}$

$x^2 = 20+\sqrt{20+\sqrt{20+\ldots}}$

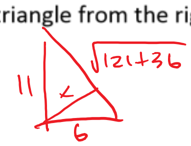
$x^2 = 20+x$

$x^2-x-20=0$

$(x-5)(x+4)=0$

$x = 5, -4$ //

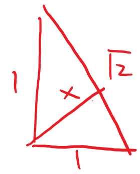
6. The height and base of a right triangle is 6cm and 11cm respectively. What is the length of the altitude of the triangle from the right-angle to the hypotenuse? Express your answer as a radical in simplest form:



$6(11) = x\sqrt{157}$

$\frac{66}{\sqrt{157}} = x$ $= \frac{66\sqrt{157}}{157}$ //

7. What is the length of the altitude of a right-isosceles triangle from the right-angle to the hypotenuse? Express your answer as a radical and ratio to the side length (height or base) of the right triangle.



$\frac{1 \times 1}{2} = \frac{\sqrt{2}(x)}{2}$

$\frac{1}{\sqrt{2}} = x$

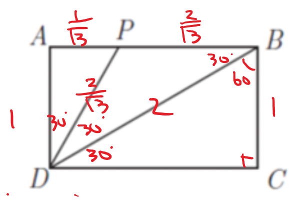
$\boxed{\frac{\sqrt{2}}{2} = x}$ //

8. For how many real values of x is $\sqrt{120-\sqrt{x}}$ an integer?

(A) 3 **(B)** 6 **(C)** 9 **(D)** 10 **(E)** 11

$\sqrt{120-\sqrt{x}} = 10, 9, 8, 7, 6, 5, 4, 3, 2, 1$ 10 CHOICES //

9. In rectangle $ABCD$, AD=1, P is on $\overline{AB}$, and $\overline{DB}$ and $\overline{DP}$ trisect $\angle ADC$. What is the perimeter of $\triangle BDP$?



(A) $3+\frac{\sqrt{3}}{3}$ (B) $2+\frac{4\sqrt{3}}{3}$ (C) $2+2\sqrt{2}$

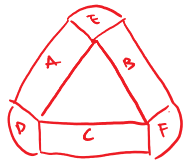
(D) $\frac{3+3\sqrt{5}}{2}$ (E) $2+\frac{5\sqrt{3}}{3}$

① THE THREE ANGLES AT 'D' ARE 30, → MAKE 30, 60, 90 TRIANGLES

② PERIMETER $= \frac{2}{\sqrt{3}}+\frac{2}{\sqrt{3}}+2$

$= 2+\frac{4}{\sqrt{3}} = 2+\frac{4\sqrt{3}}{3}$ (B) //

10. An equilateral triangle has side length 6. ~~When~~ What is the area of the region containing all points that are outside the triangle and not more than 3 units from a point of the triangle?



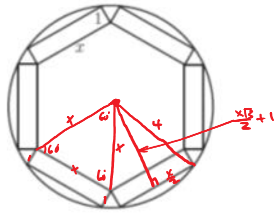
(A) $36+24\sqrt{3}$ (B) $54+9\pi$ (C) $54+18\sqrt{3}+6\pi$ (D) $\left(2\sqrt{3}+3\right)^2\pi$

(E) $9\left(\sqrt{3}+1\right)^2\pi$

① A, B, C ARE RECTANGLES 6×3
② D, E, F. ARE PIECES THAT FORM A CIRCLE W/ RAD 3.
③ AREA = $3\times(6\times3)+\pi(3)^2$
$= 54+9\pi$

B

11. A round table has radius 4. Six rectangular place mats are placed on the table. Each place mat has width 1 and length x as shown. They are positioned so that each mat has two corners on the edge of the table, these two corners being end points of the same side of length x. Further, the mats are positioned so that the inner corners each touch an inner corner of an adjacent mat. What is x?



(A) $2\sqrt{5}-\sqrt{3}$ (B) 3 (C) $\dfrac{3\sqrt{7}-\sqrt{3}}{2}$ (D) $2\sqrt{3}$

(E) $\dfrac{5+2\sqrt{3}}{2}$

① SINCE IT IS A HEXAGON, THE TRIANGLE CREATED IS AN EQUILATERAL TRIANGLE.
② CREATE ANOTHER RIGHT TRIANGLE USING THE RAD.

$\left(\frac{x}{2}\right)^2+\left(\frac{x\sqrt{3}}{2}+1\right)^2=4^2$

$\frac{x^2}{4}+\frac{(x\sqrt{3}+2)^2}{4}=\frac{4(4^2)}{4}$

$x^2+x^2(3)+4\sqrt{3}x+4=64.$

$4x^2+4\sqrt{3}x-60=0$

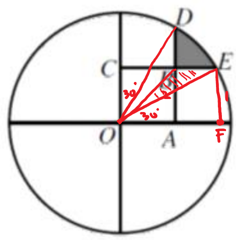
$x^2+\sqrt{3}x-15=0.$

$x=\frac{-\sqrt{3}\pm\sqrt{3+4(15)}}{2}$

$x=\frac{-\sqrt{3}\pm\sqrt{63}}{2}$

$=\frac{-\sqrt{3}\pm3\sqrt{7}}{2}=\frac{-\sqrt{3}+3\sqrt{7}}{2}.$ C

12. A circle of radius 2 is centered at O. Square $OABC$ has side length 1. Sides $\overline{AB}$ and $\overline{CB}$ are extended past B to meet the circle at D and E, respectively. What is the area of the shaded region in the figure, which is bounded by $\overline{BD}$, $\overline{BE}$, and the minor arc connecting D and E?



(A) $\frac{\pi}{3}+1-\sqrt{3}$ (B) $\frac{\pi}{2}(2-\sqrt{3})$ (C) $\pi(2-\sqrt{3})$ (D) $\frac{\pi}{6}+\frac{\sqrt{3}-1}{2}$

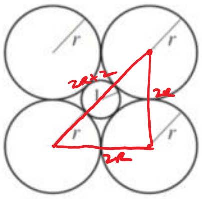
(E) $\frac{\pi}{3}-1+\sqrt{3}$

① △OAF IS A 30-60-90 TRIANGLE
② AREA = QUARTER CIRCLE $\left(\frac{1}{3}\right)-2(\triangle OBE)$ $=\frac{\pi}{3}-\sqrt{3}+1$

$=\frac{\pi(2)^2}{4}\left(\frac{1}{3}\right)-2\left(\frac{(\sqrt{3}-1)\times1}{2}\right)$

$=\frac{\pi}{3}-(\sqrt{3}-1)=\frac{\pi}{3}-\sqrt{3}+1$ A

13. A cirlce of radius 1 is surrounded by 4 circles of radius r as shown. What is r?



(A) $\sqrt{2}$ (B) $1+\sqrt{2}$ (C) $\sqrt{6}$ (D) 3 (E) $2+\sqrt{2}$

① CONNECT THE CENTERS TO MAKE A RIGHT TRIANGLE.
② HYPOTENUSE $= 2R\sqrt{2} = 2R+2.$

$R\sqrt{2}=R+1$

$R\sqrt{2}-R=1.$

$R(\sqrt{2}-1)=1$

$R=\frac{1}{\sqrt{2}-1}$

$R=\frac{\sqrt{2}+1}{(\sqrt{2}-1)(\sqrt{2}+1)}$

$=\frac{\sqrt{2}+1}{1}$

B

14. Which of the following is equivalent to $\sqrt{\dfrac{x}{1-\dfrac{x-1}{x}}}$ when $x<0$?

$\sqrt{\dfrac{x}{\frac{x}{x}-\frac{x-1}{x}}}=$

$=\sqrt{\dfrac{x}{\frac{x-x+1}{x}}}$

$=\sqrt{\dfrac{x^2}{1}}$

(A) $-x$ (B) x (C) 1 (D) $\sqrt{\dfrac{x}{2}}$ (E) $x\sqrt{-1}$

B

$=x$

15. Find the sum of the expression without a calculator:

$$\frac{1}{3+2\sqrt{2}}+\frac{1}{2\sqrt{2}+\sqrt{7}}+\frac{1}{\sqrt{7}+\sqrt{6}}+\frac{1}{\sqrt{6}+\sqrt{5}}+\frac{1}{\sqrt{5}+2}+\frac{1}{2+\sqrt{3}}$$

① RATIONALIZE EACH TERM.

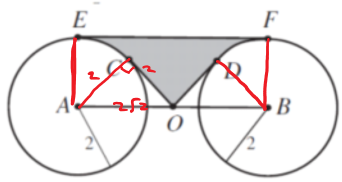
$$\frac{1}{3+2\sqrt2}\cdot\frac{(3-2\sqrt2)}{(3-2\sqrt2)}+\frac{1}{2\sqrt2+\sqrt7}\cdot\frac{(2\sqrt2-\sqrt7)}{(2\sqrt2-\sqrt7)}+\frac{1}{(\sqrt7+\sqrt6)}\cdot\frac{(\sqrt7-\sqrt6)}{(\sqrt7-\sqrt6)}\cdots\cdots\frac{1}{(2+\sqrt3)}\cdot\frac{(2-\sqrt3)}{(2-\sqrt3)}$$

$$\frac{3-2\sqrt2}{9-8}+\frac{2\sqrt2-\sqrt7}{8-7}+\frac{\sqrt7-\sqrt6}{7-6}+\cdots\cdots\frac{2-\sqrt3}{4-3}$$

$$=3-\cancel{2\sqrt2}+\cancel{2\sqrt2}-\cancel{\sqrt7}+\cancel{\sqrt7}-\cancel{\sqrt6}+\cancel{\sqrt6}+\cdots\cancel{2}-\sqrt3$$

$$=3-\sqrt3 \,//$$

16. Circles centered at A and B each have radius 2, as shown. Point O is the midpoint of $\overline{AB}$, and $OA=2\sqrt{2}$. segments OC and OD are tangent to the circles centered at A and B, respectively, and $\overline{EF}$ is a common tangent. What is the area of the shaded region $ECODF$?



① △ACO & △BDO ARE 45, 45, 90 TRIANGLES

② SECTORS ACE & BDF FORM QUARTER OF A CIRCLE.

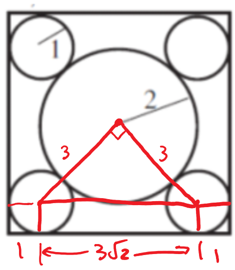
③ △ACO & △BDO FORM A SQUARE (2×2)

④ $\text{AREA}_{ECODF} = \text{RECT}_{ABFE} - \text{QUARTER CIRCLE} - \text{SQUARE}$

$$=(4\sqrt2)(2)-\frac{4\pi}{2}-4$$

$$=8\sqrt2-\pi-4 \,//$$

17. Four circles of radius 1 are tangent to two sides of a square and externally tangent to a circle of radius 2, as shown. What is the area of the square?



① DRAW A RIGHT TRIANGLE BY CONNECTING THE CENTERS OF THE CIRCLES

↳ ISOSCELES RIGHT TRIANGLE. 3, 3, 3√2

② SIDE LENGTH OF SQUARE WOULD BE $2+3\sqrt2$ //

③ AREA $=(2+3\sqrt2)^2$

$$=(2+3\sqrt2)(2+3\sqrt2)$$

$$=4+6\sqrt2+6\sqrt2+(18)$$

$$=22+12\sqrt2 \,//$$